Experiment 201-9

Damped Oscillations

Introduction

A simple mechanical analogue of a damped, oscillating system is a playground swing. If you apply an impulse to the swing, or if you displace it from its equilibrium position and then release it, it will begin to oscillate. But it won't keep oscillating forever — eventually it comes to rest because its kinetic energy is dissipated through air resistance, mechanical friction, etc.

A similar situation occurs in a circuit containing an inductance, a capacitance, and a resistance. If an electrical impulse is imparted to such a circuit, an oscillating electrical current will result, as the energy of the impulse is alternately stored in the magnetic field of the inductor and then in the electric field of the capacitor. As the current oscillates between the inductive and capacitive components, the energy imparted to the system by the impulse is dissipated through the resistive component. The charge on the capacitor, q as a function of time, t is given by

$$q = q_m e^{-Rt/2L} \cos(\omega' t + \phi), \tag{9.1}$$

where q_m is the maximum amplitude of the charge, R and L are the effective resistance and inductance in the circuit, ω' is the angular frequency of the oscillations, and ϕ is an arbitrary phase constant. However, since $q = CV$ for a capacitor, we can rewrite equation (9.1) in a more useful form:

$$V = V_m e^{-Rt/2L} \cos(\omega' t + \phi). \tag{9.2}$$

The angular frequency of the oscillations can be expressed in terms of the inductance, capacitance, and resistance as

$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\,. \tag{9.3}$$

Three physically distinct situations arise depending on whether the term under the square root sign is (1) positive, (2) zero, or (3) negative.

1. In the first case, the energy dissipated per oscillation is small relative to that imparted to the circuit. <u>Damped oscillations</u> are observed.

2. In the second case, the cosine term in equation (9.2) becomes a constant (cos ϕ), so that variation in the voltage is due solely to the exponential term. The voltage reaches its equilibrium value in the minimum amount of time, and does not oscillate. The system is <u>critically damped</u>.

3. In the third case, the circuit responds very slowly to the voltage impulse because of the relatively large resistance. The system is <u>over-damped</u>.

Purpose

The primary objective is to determine the frequency of the damped oscillations in an *RLC* circuit. The observed value will be compared to that predicted from the resistance, inductance, and capacitance in the circuit.

A secondary objective is to determine the effective resistance of the circuit.

Procedure

Before starting to work on this experiment, refresh your memory on the operation of the oscilloscope and the function generator. Refer to Experiments 4 & 8 for instructions on their use.

Setting Up: Connect the circuit shown in Figure 9.1 using the 0.1 μF capacitor. The signal generator should be set to produce a 100 Hz square wave with the OUTPUT LEVEL set to 50%. Set the time base on the oscilloscope at 2.5 ms/cm and the voltage scale at 5.0 V/cm. Check that the oscilloscope is set to trigger on CH 1, the coupling is set on DC, and the probe is set on 1X. Adjust the output of the signal generator and tune the variable resistance until the signal displayed on the oscilloscope is similar to that pictured in Figure 9.2.

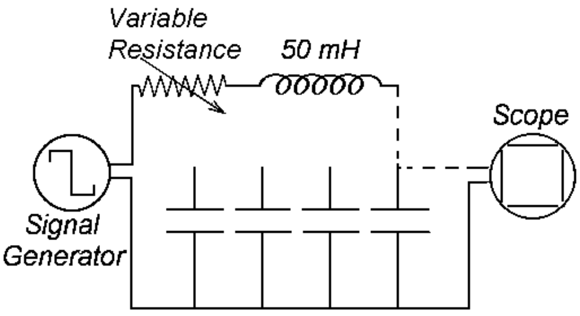
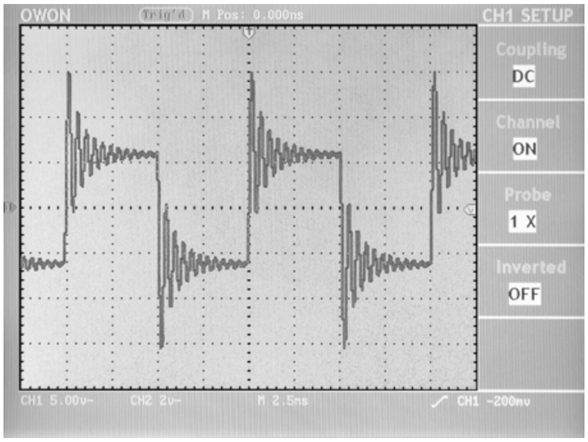


Figure 9.1 *RLC* Circuit Diagram

Preliminary Observations: Vary the frequency of the square wave from the function generator, and note what happens to the signal displayed on the oscilloscope. Then, with the frequency reset to 100 Hz, vary the resistance from 0 Ω to its maximum value. Note that the signal displayed on the scope traces the response of the RLC circuit to the voltage impulse provided by the function generator every half square wave. Sketch the trace for each of the three cases, damped, critically damped, and over-damped oscillations in your lab record book.

Damped Oscillations: With the resistance set to minimum, adjust both the Volts/Div and the Sec/Div settings on the scope so that just one of the "ringing" patterns fills the display. Adjust the position of the pattern vertically so that it's baseline is symmetric about the horizontal axis. To ensure that the baseline is accurately positioned, temporarily damp out the oscillations (by increasing *R*) so that the trailing horizontal portion of the square wave is observable.



M Pos: 0.000ns

Coupling DC

Channel ON

Probe 1 X

Inverted OFF

CH1 5.00v~ CH2 2v~ M 2.5ns CH1 -200mv

Figure 9.2 Damped Oscillations

Use the cursors to record the coordinates for each peak under Delta in the cursor menu. Time coordinates should be measured relative to the location of the first peak, while voltage measurements should be measured relative to the baseline of the signal. (Voltage and time measurements will have to be done separately.) You can use both the peaks and the valleys, but if you do, remember that they are spaced at half-cycle intervals, and only the absolute value of the voltage should be used. Repeat this for each of the remaining three capacitors (0.047 μF, 0.022μF, and 0.01μF).

Analysis

For each of the four capacitors:

1. Determine the frequency of the oscillations directly from the data. (Note that if you recorded the locations of both the peaks and the valleys, you sampled the damped oscillations every half period. Be sure to take this factor of two into account when calculating the observed frequency of the oscillations.)
2. Plot the natural log of the voltage ($\ln V$) versus time on semi-log graph paper. The resulting graph represents the exponential part of equation (9.2), with a slope equal to $-R/2L$. Use equation (9.3) to predict the frequency[1] in each case. The inductance has a nominal value of 50 mH ± 5%; the nominal capacitance values (0.100 μF, 0.047 μF, 0.022μF, and 0.010μF) are also ± 5%. Compare the predicted frequency to that derived directly from the observations. How does the frequency change with capacitance?
3. Since the variable resistance was set to 0 Ω, R represents the *effective resistance* of the entire circuit, including the capacitor and the inductor. Determine the effective resistance of each circuit from the slope of the corresponding line. Does it have a significant effect on the frequency of the oscillations? Does it change with capacitance? Provide a plausible physical explanation for your answer.

References

1. Halliday, D., Resnick, R., & Krane, K. S., *Physics, Volume 2, 5th edition, Chapter 36, pp. 830–834*

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[1] The frequency, f read from the function generator is related to the angular frequency by $\omega = 2\pi f$